Can College Students Reason?*

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> If X teaches Y, then X acts upon Y's environment in such a manner that Y develops in a desired way.
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> How Y develops in a given environment varies not only according to that environment but also according to the way Y perceives that environment.
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> Hence if X teaches Y, then X must consider not only how to act upon Y's environment but also Y's perception of his environment. Equivalently, if X is <u>not</u> sensitive to Y's perception of his environment, then X cannot be teaching Y.

Simplistic as this argument may sound, most of us who try to teach college mathematics have tended to ignore it and its implications. We rarely consider how various students perceive their learning environments--specifically, our classrooms. We also are mostly ignorant of recent research that dramatizes some important consequences of our omissions.

> <u>Item</u>. Towler and Wheatley of Purdue University asked students in an introductory mathematics course whether or not changing the shape of a clay ball affected a) the amount of clay, b) the weight of the clay, or c) the amount of space occupied by the clay. Although most students realized that mass and weight of the clay were invariant, 39% of them believed that the volume changed when the ball was rolled into a sausage shape.[1]

Purdue University, of course, has no monopoly on such thinking; we all have experienced the student who just can't seem to catch on to our mathematics, no matter how hard he tried, the student who can do no more than memorize how to manipulate some formulas. Moreover, in this day of opening admissions and dropping enrollments, it is unlikely that the number

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of these students in our courses will decrease. Not all these students are lazy, or dumb. Some work very hard for us, meeting only frustration; some are quite successful in other courses. Is there anything we can do for them?

I believe we can find at least partial answers to such questions if we consider a psychological theory that makes good use of the concept of the learner's perception of his environmental stimuli. The framework I wish to oversimplify for you today is that developed by the Swiss investigator Jean Piaget.[2] Although Piaget began work in the 1920's, he was largely unknown in this country until the last decade or so. His work by now has influenced elementary and, to some degree, secondary school teaching, but still has not received the attention it deserves from most college educators.

Piaget describes the mental development of a human being in terms of an undefined concept that is roughly translated as "mental structure". An individual organizes environmental stimuli according to his mental structure, and adapts this structure to assimilate such stimuli. Except for inherited reflexes, an infant's mental structure is very narrow; it can assimilate very few of the many stimuli encountered. But, given sufficient numbers of these stimuli, the structure accommodates itself for organizing a broadening range of them. A structure changes when it encounters stimuli that differ only slightly from those it can handle. If there is no incongruity, the stimuli will be assimilated without structural change; if there is too much incongruity, the stimuli will be ignored.

Although Piaget and his followers describe typical mental growth in

terms of a refined system of stages and substages, the most important observation for our purposes is that persons encounter the stages in order. They do not backtrack, and, ideally, development does not involve skipping stages, which can lead to problems later in the growing period. (In an extreme case, a special educator might take a "slow" 8- or 9-year-old back to the crawling stage, and then teach him to walk again, and so forth, gradually rebuilding his mental structures to catch up with his physical development.)

With the hope that we can separate the wheat from the chaff, which is abundant in any psychological theory, we shall concentrate here on two of the major stages--those of "concrete operations" and "formal operations". Piaget uses the term "concrete operations" to refer to an extended period between the approximate ages of 7 and 11 in which a child has become able to set up one-to-one correspondences, to count, to recognize that the number of objects in a set is independent of its configuration, and to imagine himself in the position of others. He could perform none of these operations before reaching this stage, and his aptitude improves during this stage. On the other hand, he *cannot* yet operate *on these operations* by designing an experiment that requires holding all but one variable constant, or by formulating hypotheses, or by recognizing that volume is independent of shape or weight, or by responding to the form rather than the content of a logical argument. He will probably not take a fastidious interest in the rules of games. Ability to perform these operations on operations, or "formal operations", is acquired around the age of 11 or 12, according to Piaget.